Equality at the Top of the Corporation:

Assessing Possible Worlds of Mandated Quotas

Bruce Kogut*, Jordi Colomer**, and Mariano Belinky***

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*Sanford C. Bernstein & Co. Professor, Columbia Business School; **Senior Researcher, Copenhagen Business School; ***McKinsey.

Abstract

A firm is conventionally defined as an economic entity, but it is also a social community that is often subject to broader debates regarding social justice and its ethical commitments to possible and, for some, preferable worlds. A challenge to social science is how to analyze strategies to achieve possible worlds that do not exist. We propose a methodology to assess polices to address a stunning property of the governance of corporations, namely, the paucity of female directors to corporate boards. Using estimates from the social experiment of imposing quotas on Norwegian boards of directors, we apply an agent-based model to American board data to show that modest numerical quotas quickly generate well-connected networks of women directors who attain equality in their centrality and influence.

1. Introduction

...as there is hardly any inequality in the state of nature, ...it follows that moral inequality, authorised by positive right alone, clashes with natural right, whenever it is not proportionate to physical inequality; a distinction which sufficiently determines what we ought to think of that type of inequality which prevails in all civilised, countries... -Jean-Jacques Rousseau, Discourse on the Origins and Foundations of Inequality Among People. (modified from the translation by G. D. H. Cole, public domain)

Inequality among social groups marks all human societies and is unrelated, as Jean Jacques Rousseau derided in the eighteenth century, to innate or physical ability. A distinguishing feature of inequality is its foundation in gender, racial, religious, and ethnic distinctions that perpetuate discrimination. As a result, in many countries, quotas have often been employed as a means to assure greater equality in outcome as well as in access and opportunity, though rarely in the context of the management and governance of the public or private firm.

This absence of egalitarian access to power due to gender or race is easily observed and much condemned and yet hard to change. In the past decade, a silent revolution has occurred insofar that many countries now mandate gender quotas for political elections[1]. Whereas quotas once addressed religious (e.g. in Lebanon) or ethnic (e.g. in Malaysia) equality, more recent ones have sought to create more equality among the sexes.[2]

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[1]See the web site: http://www.quotaproject.org/index.cfm

[2] See Beaman et al, 2009, for a study of Indian villages and quotas.

Being part of society, corporations and their governance also reflect broad social inequalities.

Regulatory interventions to correct such inequalities are rarer in the private than public sphere, and yet there are many notable exceptions. For example, the Norwegian law, passed in 2003, stipulated that the boards of directors of all public and state-owned firms must consist of at least 40% women directors by 2008.[3] This law represents a radical extension of quotas from the political arena to the private economy. Failure to conform invokes serious penalties, including the threat of closure; by 2008, Norwegian public companies were reported to have complied with the legislation. Other countries (e.g. France, Spain) have followed suit.

The motivations for quotas are not hard to understand. For the United States, the proportion of women chief executive officers is less than 3%, and the proportion of female directors has been historically under 15%.[4] With very few exceptions, the proportions for other countries are similar, if not lower.

Legislating outcomes by quotas focuses on the relative proportion of the in- and out-groups (e.g. men and women). The persistence of inequality indicates that the process of selection is tilted towards favored groups, often through club-like social ties that rely upon distinctions of gender, race, religion and other social markers. However, an ambitious quota is often seen as an unfair and costly intervention that discriminates against the

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[3]The law is described at the web site of the Norwegian Ministry of Children and Equality: http://www.regjeringen.no/en/dep/bld/Topics/Equality.html?id=1246 .

[4] Calculated from the Execucomp data in the Compustat database.

dominant group. The tension in the use of quotas stems from this contradiction in the use of quotas to redress social injustice for one class of individuals through decreasing the opportunities for another class.

Thus, the debate on diversity, formerly labeled under 'affirmative action', has historically been riddled by conflicting political philosophies and jurisprudence traditions pitting concepts of consequentialist fairness and libertarian values.[5]. Not surprisingly, in most countries and certainly in the US, courts and legislatures have not interceded in the social composition of top management and boards, whereas labor law for many other positions has sought to regulate employment practices. Consequently, while firms have greatly expanded their corporate social responsibility practices in recent years and have sought to increase the diversity of their workforces, they have generally stayed clear of imposing diversity quotas on the composition of their managers and boards of directors.

Arguably a more elegant and political feasible solution than aggressive quotas is to improve egalitarian access and to permit individual choice unencumbered by such quotas to determine the proportionality of representation. But how can such a state of affairs be realized in the absence of egalitarian access? The alternative solution of a modest quota, we will argue, relies on a passing of a critical threshold that permits minority groups to attain sufficient access into the selection process.

The passing of a threshold is not a new idea to studies on gender and governance. Based on her ethnographic studies, Rosabeth Moss Kanter (1977) proposed that the proportion of managers who are women becomes stuck at low levels since a 'critical mass' is rarely achieved; she envisaged that the 'tipping point' to be at 1/3rd female participation. A

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[5]For a philosophical treatment of these issues, see chapter 7 in Sandel.(2009)

high quota succeeds by tipping the equilibrium past this critical state. Because the crossing of this tipping-point threshold is often not achieved without quotas, minorities are dependent upon selection by the dominant group, fostering the persistence of inequality. This type of equilibrium is a reflection of *sub-criticality*; the density of women is too low to generate enough endogenous pressure for equality at current levels.

Therein lays the social science argument for a quota. However, how big should this quota be? The size of the minimal quota to tip the equilibrium will depend upon the process by which directors are chosen. Evidence indicates that directors and top managers are often chosen through personal and professional networks by using particular *rules*, such as acquaintances, gender and ethnicity, and educational backgrounds (Davis et al, 2003; Mizruchi, 1996; Stafsudd, 2006). A recent study found that board interlock networks facilitate, for example, CEO pay, while the exact causality is due to the homophilous reasons that determine director selection in the first place (Kim, Kogut, and Yang, 2011).

This suggests a modeling approach by which the social rules for the determination of director selection in the context of director-board networks are estimated. Then we apply these rules to simulate the evolution of the director network under the imposition of gender quotas. Through grounding the rules (and their estimated parameters) in empirical data, the simulations generate possible worlds close to the observed world but with one significant difference: the gender ratio for women will have improved.

By this approach, we analyze the proposal that modest quotas can often be sufficient to improve egalitarian access to the top hierarchical positions in society. We begin with two documented presumptions, one grounded in law and the other in social practice. The first is that boards appoint not only CEOs, but also other directors. The second is that the labor market for directors is influenced by recommendations among ’friends of friends’ as well as other social influences found in human, and board interlock, networks.

These assumptions allow us to test for the minimal quota. From our theoretical lens, the low percentage of female directors reflects their sub-criticality; there is simply an insufficient number of female directors to tip director selection towards more egalitarian outcomes. The analytical methods rely upon a multi-agent model in the context of a bipartite network formed by the affiliation of directors who belong to boards. We work with the directors' one-mode projection of this graph, meaning that two directors are connected if they sit on the same board. Technically, the simulations exploit the sensitivity in networks to experience significant changes in aggregate outcomes in response to relatively small micro changes, e.g. an imposition of a low quota.

The challenge confronting research into questions on social justice is simply the data are available for existing worlds, not possible worlds. A simulation approach permits this investigation. Gender, racial, and other forms of inequality are the norm, and empirical work has been limited by the infrequency of gender-balanced organizations. The challenge to simulations is to generate a plausible degree of external validity to worlds that do not exist. The methodological proposal we follow is to ground the analysis in data from one natural experiment (Norway) to an *untreated* case of the US. This approach provides guidance to interventions, such as gender quotas, to inform policies of corporate social responsibility and of public interest, if not more broadly to interventions involving more classical questions of competitive strategy.

2. Equality and Quotas

Competing conceptions of equality and social justice have a long history, especially in philosophy and law. Governance of corporations has hardly been immune to these debates.

In the United States, quotas have been at the core of active litigation in relation to affirmative action and positive discrimination. While these debates are complex, it is useful to note the contrasting definitions of equality used in this paper.

A quota is a percentage target that mandates a proportional representation of a particular group which is often but not always disadvantaged. Operationally, affirmative action can stipulate that a property of individuals, such as race, be a factor in selection, but without stating a specific quota. Alternatively, affirmative action can be realized simply by mandating the global target and leaving the rules of selection to the choice of the impacted organization and individuals. Generally, the former is used in affirmative action for educational admittance, as in the famous U.S. Supreme Court case of Grutter v. Bollinger; the latter is the application used in the case of Norway.[6]

The Norwegian quota was either onerous or distasteful to many private property owners. The sheer mechanics of engineering 40% targets reveals the awkwardness of the law. For small organizational entities such as boards of directors, quotas are sensitive to so-called ’integer’ problems; consequently, the Norwegian law had to lay out precisely the rule when to round up or down, depending upon the board size. (We find that these problems matter to the simulation results, as discussed later.) The number of public limited companies fell by 23% after 2003 (when the law was passed) and there was a four time rise

[6]According to Sturm (2006, fn.10), in the Grutter v. Bollinger case, the Supreme Court in 2003 determined that the admissions plan of the University of Michigan Law School did not violate equal protection, based on a holding that diversity is a compelling interest that can justify the narrowly tailored use of race in selecting applicantsf or admission to public universities. The same year in Gratz v. Bollinger, the Supreme Court ruled against an admissions policy that assigned points to applicants considered to be diverse. The court has not accepted diversity as a factor in private hiring decisions.

in the number of Norwegian firms who chose to incorporate in London, an increase far greater than from other Scandinavian countries (Ahern and Dittmar, 2009). Ahern and Dittmar (2009) estimated the overall performance of Norwegian firms to have been negatively affected by the quota; however, the negative impact was due to the age and inexperience of the new directors as opposed to their gender. Given these economic effects, the concerns over the private costs of social justice are not to be easily dismissed.

A possible resolution to the impasse due to insufficient egalitarian access for a minority and yet the dislike of aggressive quotas is to consider a relatively modest quota. John Rawls (1999) proposed a consideration of fairness to redress the perennial liberty and equality stalemate, whereby a social intervention should be consistent with the interests of the least well off. This proposal does not advocate a concept of global equality, but an articulation of fairness in deference to ameliorating the opportunities of the least privileged. The debate remains whether such a proposal can justify the private costs. A good rule might be to ask what policies affect justice, not only at a minimal cost but with minimal disruption to existing social distributions. A good example of this kind of policy is suggested in the Bowen and Bok study on positive discrimination in college admissions that transformed this debate between affirmative action and liberties by showing that small deviations from blind review can result still in large increases in social equality, however at a private cost to some (Bowen, Bok, et al., 1998).

Can such a result be found for quotas? Clearly, a lower and more modest quota is politically and socially more acceptable, especially in reference to the private economy. Counter intuitively, low quotas can often also be very effective in improving the position of disfavored groups by subtly increasing the connectivity among minority members, thereby increasing equality in outcome and in access. In cases where nominations to positions of power rely upon social relations between members of clubs (e.g. ’old boys’ clubs’), low quotas can generate not only greater representation but also more powerful representatives belonging to the affected group.[7]

Quotas are more than simply a counter-balance to existing discrimination, but also a mechanism to break endogenously the beliefs that favor the inequality in the first place. Most of the prior literature on ’critical mass’ has focused on the changes in beliefs due to increases in women participation (Studlar and McAllister, 2002). We propose that quotas can also change the structural properties of power and its distributions among groups. This change is realized through the possibility that quotas will enhance the importance of actors belonging to the affected groups and to the relations among them, permitting a self-organizing new equilibrium. This result depends upon the structural properties of the director network.

3. Tokenism, Homophily, and Structure

The work on gender and top management has consistently noted the importance of structure and below-criticality as explanations for the persistence of low representation by women and minorities on boards. While quotas have pushed Norwegian boards to greater than 40% of directors to be women, the percentage of women who are the board chair and the CEO is less than 5%. Female representation on boards of directors has stagnated, on average, at less than 15% in most countries in the European Union and is only marginally

[7] Studies of gender quotas for political representation find low evidence for a ’critical mass’, but suggestive evidence that low quotas produce a larger marginal gain in representation than large quotas(36). A review of the studies on critical mass theory is found in Childs and Krook (9).

higher in the United States, suggesting that boards do not move beyond a symbolic gender (and racial) representation.

Symbolic female representation constitutes what Kanter called a ’token’ response to external pressures (Kanter, 1977). Selection by tokenism runs counter to homophily and has a potentially pernicious effect: such an exogenous policy does not generate the social networks that would endogenously sustain female representation. Tokenism means that the nomination of relatively few women to boards relies upon a symbolic response as opposed to a selection relying upon endogenous social networks that self-replicate if connectivity is high. Self-replication could be achieved through homophilous social conventions, by which people of ’like’ traits nominate similar people (Goodreau et al, 2008). If, though, a group is below a critical threshold (i.e. minority members are too few and poorly connected), homophilous rules will not be effective. This observation is strengthened by Peter Blau’s observations of the structural effects of large size differences between majority and minority groups, the latter tending proportionally to have higher contacts with the majority than the majority with the minority (Blau, 1977).

Management studies have long found evidence for tokenism and gender segregation. Harrigan (1981) early noted that the distribution of female executives reveals a strong industry pattern -the distribution is not simply random. Using more recent data, Kogut and Yang (2012) find that the industry distribution of female CEOs is not random. Moreover, this pattern is self-replicating due to the tendency of boards to replace exiting female directors by women. Farrell and Hersch (2005) found that women are added to boards when a board has low or no females and for the replacement of exiting women. Adding a director, therefore, is clearly not gender neutral.

The puzzle of why there are so few women board members applies more strongly to paucity of female CEOs. Dailey et al. (1999) observed that not only were there few female CEOs in the late 1990s; there was also few in top management in general. In a notable study, Lee and James (2007) note that the number of women managers increased from 18% in 1972 to 45% in 2000, and yet were less than 1% of CEOs in 2000, according to Bureau of Labor Statistics and Catalyst data. Their results showed that stock market reactions were negative for the announcement of female CEOs. While Lee and James noted company and CEO experience closed this gap, the difference in (negative) returns remained significant for new male and female CEOs. As discussed later Ahern and Dittner (2010) also find important particular characteristics, namely age and the lack of experience, to account for the gap in Norway. The findings of Lee and James imply that the market holds low expectations about female CEOs ability. Bias is both statistical (the market holds lower opinions for female CEOs) and a preference (or taste) for male CEOs, for the negative assessment holds even after controlling for specific individual characteristics, such as experience.

We might want to know if such assessments are warranted. Wolfers (2006) seeks to supply an answer by looking at the excess returns associated with holding shares in companies led by women and then compares these returns to a portfolio consisting of shares in companies headed by male CEOs. The female portfolio fared somewhat worse. However, since the number of female lead companies is so small, the estimates are surely sensitive to outliers and noise. The broader issue raised by this study is if the difference between male and female-lead companies is nearly statistically zero, why are there so few women CEOs.

The study by Kogut and Yang (2012) found that women directors and CEOs are found more in some industries than in others, as noted above, but also boards with more women are more likely to hire female CEOs. Possibly, there is assortative matching going on, with joint effects of industry type, women directors on boards, and female CEOs. Whatever the causality, the assortativity is the central observation, though there are also industry effects too (some industries have more women). While there was no evidence women CEOs were paid less once controlling for industry effects, pay increases were largest when a man replaced a female CEO, and lowest for the converse. But in all, the Kogut and Yang study suffers from the low count of women CEOs, as do all studies on this question.

While concerns over the low representation of women are found in many domains, including scientific research, there is a substantial empirical literature that analyzes the specific relation of networks to the career prospects of women in management.[8] These studies indicate strong gender stratification by job and industry, but find mixed evidence for tokenism and homophily (Petersen et al., 2000; Reskin and Bielby, 2005). More recent studies find homophily operative at the level of job referrals (the supply side) but is absent or weak at the level of job offers (the demand side). (See Fernandez and Fernandez-Mateo, 2006).

Contrary to these studies on the surprising low level of bias in internal labor markets, research on corporate top management and boards has consistently found tokenism and dominant male homophily (Stafsudd, 2006). In other words, bias appears more at the top than at the bottom. But what then is the difference between top and low management recruitment? In her study on high-skill contract work, Fernandez-Mateo (2009) points to the play of a 'cumulative gender disadvantage' in wages, which is partly explained by the higher mobility of male contract workers but also unobserved perceived traits on the

[8] See Fernandez-Mateo, 2009: 875-77 for a review. On women and science, see (Etzkowitz, 1994; Ding et al., 2006).

demand side, which might be simple bias. Whatever the motivation, the dynamics of such a labor market results in stratification and differential career advancement: males are promoted and paid accordingly higher wages. This study then poses the puzzle that whereas there may be low gender differentiation in wages within a job category, dynamically career advancement to top position are influenced by demand factors.

Several early studies point to deficiency in women networks at the top of the corporation as the resolution to this puzzle. In one of the first statistical studies on social networks, gender and managers, Ibarra (1992) found that work relations among men were more homophilous. Women's personal networks were also homophilous, but women relied heavily upon male networks for instrumental goals. The heterophilous patterns of women managers reflected the strategy of a minority group below a critical threshold, forced to rely disproportionately upon the male managerial majority. This situation is what we mean by 'sub-criticality'. This strategy comes at a cost. Burt (1998) found that women rely upon 'borrowed social capital' of their usually male boss, which would be an example of an instrumental strategy. This strategy resulted in lower chances for promotion and lower pay. In conjunction with the Ibarra findings, these results suggest that instrumental networks may in fact be limited for advancement. This implication is surely consistent with the lower percentage of female top managers.

The study by Cohen et al. poses explicitly the question of how can women gain entry into top managerial positions in the absence of homophilous social networks and at sub-critical proportions (Cohen et al., 1998). They found that women are less likely to be promoted to positions where they are not already present. They conclude that it may be that what is thought of as a glass ceiling is actually a glass door, which can only be opened by women if other women have opened it previously. If so, patterns of sex segregation in managerial ranks are unlikely to change drastically through processes endogenous to employing organizations.

Board selection and gender have been less studied, leaving aside the demographic assortativity noted above regarding the correlation of CEO gender and the proportion of women on a board. Westphal and Zajac (1995) point to the tendency of boards and CEOs to be demographically similar; while gender was not one of the characteristics presumably because women CEOs and directors were especially sparse at the time of the study, it points to the importance of social rules, such as education, age, and style, as factors in choice. Westphal and Milton (2000) showed that minority board members can achieve comparable levels of influence to majority board members through sharing memberships on other boards.

Overall, the US board structure is marked by a 'small world' topology in which directors belong to local clubs which are loosely tied to each other (Davis, Woo, and Baker, 2003; Conyon and Muldoon, 2006; Kogut, 2012). The choice of directors into these clubs is driven by homophily (e.g. gender), popularity (or experience), and current membership in a club (which in graph terms their local neighborhood clusters). These properties are indicative of *social rules* by which directors are invited onto boards, thereby causing the board-director graph to evolve through the formation of new links and nodes. These rules become the foundation of our analysis for the estimations and simulations.

The above findings lead to the following conclusions and proposition. The tendency for female percentages to plateau represents the satisfaction of a token objective by many even if not all boards. Once token representation is achieved, the density of female directors is too sparse and disconnected to bootstrap to higher levels of participation by homophilous relations. Consequently, the female network (i.e. subgraph) remains subcritical. Absent stronger external pressures, the percentage of women on boards represents a plateau given current societal norms and the weak endogenous mechanisms for reform. This leads to the proposition that quotas may be useful in minimally offsetting the negative endogenous mechanisms that favor men and may more positively tip recruitment networks towards self-organizing criticality to support great female CEO selection.

4. Terms and Definitions

The central theoretical property of interest is the concept of structural equality. Simple numerical equality is the equality in the number of female directors. However, even this simple definition confronts a dilemma which belies the motivation for the subsequent analysis. Does numerical analysis mean only that each board should attain at least 40% directors, or does it mean that 40% of all directors should be women? Clearly, some women directors may sit on multiple boards and thus the total number of women can be less than 40% if women directors have a higher degree than men, that is, they sit proportionally on more boards. In this case, female directors should have higher average degree, namely, they should be connected to more directors than their male counterparts.

Let's take this to the extreme and consider a director network that has 10 female directors and 90 male directors. The ten women sit on 9 boards each and the men sit on 1 board each. In this case, who is more powerful and which sex is advantaged? Women and men both hold 90 seats, but an individual woman has a position of structural power that is far more significant than a male director.

An obvious candidate definition of power in this context is to look at network centrality of women and male directors. We focus on 'betweenness' centrality that measures the role importance of brokers acting as intermediaries. Betweeness centrality is calculated as the number of geodesics (that is, the shortest path among any two directors) that passes through a given director.[9] In this sense, the more directors that depend on that director to communicate efficiently with others, the more power she or he has. Betweeness centrality is then an intuitive measure of structural equality", since an increase in average female centrality indicates greater opportunities for brokerage by women directors. Given the empirical heterogeneity in degree (the number of directors to which a given director is connected), average betweeness centrality is unrelated to other notions of equality, such as individual fairness by which all individuals have equal influence and opportunity.

Quotas are in many ways costly. Since they are politically contested, resources are expended to implement them and to resist them. Moreover, the efficiency gains of quotas are ambiguous. Either by greater diversity leading to better decisions, or by providing opportunities to qualified candidates otherwise shut out, quotas can improve the efficiency of a firm.[10] At the same time, quotas might also eliminate male directors who may be better than less experienced female directors. This indeed was the finding for the Norwegian experiment as discussed earlier.

What would be desirable then is to impose the minimal quota that is sufficient for achieving a critical mass of female directors such that they do not need to rely upon male

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[9] See Wasserman and Faust, 1994. Mizruchi and Potts, 1998, provide an analysis of centrality and power through simulations that emphasizes the importance of topology.

[10] See the provocative book by Scott Page (2007) on the efficiency benefits of diversity –though he remained silent on how to achieve such diversity, e.g. quotas; this view deviates from the more traditional economic focus on a preference (or taste) for bias or for statistical error (sometimes called an ecological fallacy) in which individuals are categorized (e.g. white, black) and accorded these demographic statistical averages.

directors for intermediation. The most powerful findings from previous studies of the relation of network positions and power is the implication of 'sub-criticality'. In network terms, the signatures for sub-criticality will be a low level of women to women connectivity that will be evident in a small *giant component*, that is, the size of the largest sub-graph consisting of connected women directors who can be reached by a walk.

The imposition of a quota addresses an implication in the literature on the importance of quotas to disrupt a low equilibrium of inequality and to move structurally the system to a more equal outcome. Ideally, we would like to define a point of criticality or critical mass in which the women director density is sufficiently high that it self-organizes (2). Self-organization in this context means that once a critical mass is attained, a high level of gender equality is maintained even when quotas are removed, an exercise we implement below. Nor is this exercise of stability without empirical relevance, for the removal of gender quotas in Egypt and Pakistan lead in both cases to substantial declines in female representation (Paxton et al., 2010).

A priori, we don't know the tipping point to self-organization for complex social networks. We implement consequently a straightforward analysis of critical mass by removing the quotas after a number of iterations and verify if the network properties are stable or if gender bias increases. A society that is perfectly indifferent to gender would not reveal any patterns of homophily. Homophily results in a graph in which *like* people are located in groups that have a high within density and few links to the other groups; a heterophilous society would show higher between links than expected by the population proportions.

These observations lead to the strategy for measuring homophily of a population through the concept of modularity homophily (Newman and Girvan, 2004). Directors are assigned to one of two partitions (male or female). By calculating the within- and between-links of these modules, the modularity statistic indicates if director ties are homophilous or heterophilous. A positive modularity statistical indicates homophily whereas a negative statistic indicates heterophily. A statistic close to zero indicates no first-order effects of gender on director ties, such as would be found in a random network.[11]

5. Data and Descriptive Statistics

Norwegian Data

As described earlier, Norway imposed in 2003 a law imposing a quota of 40% on all public and state enterprises to be reached by 2008. We collected bipartite network data for the years of 2002 to 2008.[12] The data size is similar to that of Ahern and Dittmar (2009), except that many financial institutions were removed in our data; thus our data size is slightly lower.

Because of some inconsistencies in these data, we further filtered the data by the following process. The goal was to estimate the weights to the arguments in the utility functions of directors and boards in Norway that best fit observed transition between networks of consecutive years. We transform consecutive networks so that they contain

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[11] We also tried another approach, which proved less informative, using a formula developed for non-parametric distributions of degrees to calculate this critical ˜threshold" where network connectivity begins to disintegrate. We used the concepts introduced in Callaway et al. (2000) that provides the equation $q_c = \frac{1}{G_1{}'(1)}$ to estimate the number of nodes that can be removed from the network without destroying the giant component, given the degree distribution of the network.

[12] Latapy et al. (2008) gives an excellent exposition on bipartite director/board graphs.

exactly the same nodes and only the edges change. By taking the union of the directors and the intersection of boards, only boards appearing in both years will also appear in the filtered network, whereas a director that appears in one year will appear in both networks. If we find that a director was a member of a board in the previous and subsequent years but not in the current one, we assume that this indicates missing data; we then add the missing edge to the network.

In table 1, we describe the gender and degree distribution statistics for Norway.[13] The letter 'k' indicates degree and is the number of boards per director. Not surprisingly, the number of women rises due to the quota, although there are fluctuations due to changes in the number of firms in the sample. (The sample sizes change according to the source of the data as well as to the exit of some firms as listed companies.) Due to the increased size of the data for 2006 to 2008, the average degree falls, because the additional firms are smaller with smaller boards and thus firms have fewer directors on their boards. More interestingly, the low average degree of the directors (male or female) indicates that the director network in Norway is not particularly social and connectivity is low. This low degree has an important implication: the percentage of women in the entire network is very close to the average percentage per board. Thus, the Norwegian board is very disconnected compared to other board networks, including those in other Scandinavian countries (Edling et al., 2012).

The most distinctive deficiency in the position of women directors in Norway is the

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[13] The earlier data were provided to us by Alexandra Gregoric (Gregoric et al., 2009) , whereas the last three years came from the official register of all public firms in Norway (http://www.brreg.no/english/). The difference in the number of firms between the two sources was corrected through a process described below.

fragility of their connectivity. The percentage of women belonging to the largest component of the director graph is smaller than the men's, indicating that the female entry did not generate high connectivity. The rapid increase of Norwegian female directors relied upon a process that matched outside candidates to boards; the nominations were largely of new directors, with over 150 appointments made via a government sponsored headhunting firm.[14] This selection rule generated very low connectivity, which resulted in a poorly connected female graph. Thus, the average centrality of women relative to men actually fell between 2003 and 2008, though was still two times higher than average male centrality since women had slightly higher average degree (i.e. multiple board positions).

Notably, though, the connectivity is not much higher in 2008. The Norwegian case indicates the importance of the initial topology of the network as we know from other studies (Watts and Dodd, 2007; Tassier and Menczer, 2008); the more anomic and low connected Norwegian network reveals far less evidence of clubs and powerful directors found in the US. By 2008, women had vastly increased their share of board seats, but their sub-graph is substantially more disconnected than that of the men, since their assignment was random.

This fragmentation is clearly apparent from Figure 1 comparing the Norwegian empirical networks for 2003, 2005, and 2007 in the left column. The figures are visualizations of the *bipartite* networks; these figures show boards at the center of a configuration where the outer circle consists of directors and links to other boards indicated shared directors. There are many boards that are *isolates* and are not linked to others or which are situated in rather small components. The isolates act as new entrants into the

[14] Personal communication, Elin Hurveness.

network when they become connected to other boards. The right column shows the network once it was filtered by the rules we discussed above. This filtering eliminated the variance in network sizes from year and lead to more stable estimates.

US Data

The US data for this project were obtained from BoardEx for the year 2007.[15] As seen in Table 2, the US data consist of 6,519 U.S. firms and 29,750 directors, occupying 40,356 board seats; the average degree (i.e. the number of boards to which a director on average belongs) is 1.35. Table 2 also indicates that the number of US directors by gender is 27,183 male and 2,487 female, thus women directors made up 8.3% of directors. The top 500 firms by market cap have a percentage of women of all directors of 9.1%.

As seen in figure 2a, the board size histogram shows that boards with seven directors is the most common. A large number of firms (~1,350) has only one director in them. Since the data are director-centric, boards of one indicate current director roles in companies outside of our sample. We remove director appointments in foreign firms, thus leaving only the U.S. firms and board members present in the data. After this removal, we are left with 29,670 directors and 4,884 firms, with a total 38,224 board seats, as reported in Table 2.

Figure 2b provides the histogram of the number of women in boards, with *zero* women being the modal case. A Poisson distribution fits the data well, suggesting that there is no statistical evidence that some firms are better or worse than others; rather, the

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[15]BoardEx LLC is an independent, privately owned corporate research company that commercializes corporate governance and balance sheet data.

primary inference is the number of women directors is so low that randomly, we should expect many boards with zero or one woman.[16]

In table 2, we also show data from other countries: Sweden, India, and Belgium.[17] The firm degree (with *k* denoting degree) varies considerably by country, indicating a wide variance in the size of boards. The percentage of women on Indian boards is about 5%, substantially below the figure of 17% for Sweden. The striking property of the Indian data is the high degree for both male and female directors: Indian directors sit on multiple boards and the network is overall very connected.

By this comparison, the Norwegian network can be seen as quite distinct in its low connectivity and director degree. The compensating virtue is the Norway provides the only natural experiment in imposing female quotas. To smooth out the variances due to changes in sampling, we analyze the filtered data as described above. It is neither possible, nor desirable to augment the Norwegian director degree to approximate that other countries, and this feature of Norwegian boards should be kept in mind as a limiting factor in the construction of the simulation modeling, described next.

6. Constructing the Simulation Model

Explorations of complex social phenomena can be greatly facilitated through the use of multi-agent simulations (Hedström, 2005; Miller and Page, 2007; Epstein, 2006).

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[16] The age distribution of directors indicates a slightly younger female director distribution compared to that of men, with a peak ratio of 17% female directors at around age 50. However, the numbers are so low that this younger cohort will not by itself substantially change the percentage of women on boards.

[17] The sources of these data are given in Kogut, 2012.

Simulations can be a form of counter-factual investigation. Our innovation is to ground the simulations in a current world (the Norwegian quota experiment) to analyze a possible world if the US should also adopt a quota. This might be seen as a type of 'discontinuity' design (the Norway quota) but projected onto the US. In all, such 'as if' investigations into existing social networks are rare and provide an example of value of computational social science to analyze the implications of public policies (Lazer, et al., 2009).

The imposition of quotas by Norway offers then a natural experiment, but the number of conditions that differs between countries will still trouble extrapolation. A possible world counterfactual follows the dictum of David Lewis that it is best to compare worlds that differ in the fewest factors (2001). This dictum is implicit in many kinds of counterfactual statistical treatments, such as propensity scoring, as well as in other counterfactual methods, such as simulation which we use here.[18] The simulation structure that we apply begins by estimating through genetic algorithms the parameters to preferences that guide the evolution of the Norwegian boards. We then apply these estimated weights to an agent-based model for the evolution of the US board gender composition whereby boards seek to recruit women in order to satisfy the quota. The use of agent-based model in strategy was pioneered in strategy by Axelrod et al (1995); we follow this approach in a different modeling context with the innovation of using actual data.

The central engine to our simulations is a rule that matches directors and boards. The evolution of the bipartite network (boards and directors) is a simple type of 'link formation game' whereby a director joins the board (a link is created) if the utility of the

[18] See Tetlock and Belkin, 1996, on the desired properties of counterfactual analysis.

director is improved and if the utility of the board is improved.[19] The board draws randomly one potential edge between board *i* and director *j* in 10 trials. For each trial, we calculate the previous utility without the link and new utility with the link for the director, i.e. *u(g+ij,i) - u(g,i).* We then calculate the utility of a board is defined as the average utility of its members with and without the new director. If the board has members a,b,c, the change in utility would be *(u(g+ij,a)-u(g,a)+u(g+ij,b)-u(g,b)+u(g+ij,c)-u(g,c))/3.* For each of the 10 director and board pairwise matches, we assign the minimum increment in utility to the pair, since the agreement has to be satisfactory to the party that least benefits. We then select the pair that has the maximum increment in utility and add the link: the director joins the board.

This method is not assured to reach a global welfare maximum, though over sufficient iterations, it should approximate a pairwise equilibrium. Other algorithms can consider side payments, or coalitions if board members should vote, or consider a maximum rule instead of the maxmin we imposed. We did not find much gain to say permitting voting by members instead of averaging and we thus chose less computationally costly..

We turn now to explaining the utility functions which are drawn from our earlier discussion on the empirical results on board selection and then to the thorny problem of estimating the parameter weights to the arguments to the utility function.

Utility function

The utility function is a weighted sum of 4 elements.

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[19] For a review of these models, especially in the context of a labor market, see Jackson (2008).

$$u_i = Wc_i \times compliance + Wh_i \times homophily + Wp_i \times popularity + Wccf_i \times CCF + Wccm_i \times CCM + Wr_i \times Random$$

We explain each of these elements in the following:

1. Quota compliance

Each board has to comply with the statutory requirement to satisfy a quota of women on the board. The cost of this compliance is higher for those boards which were least in compliance at the start.

Average of the cost of the boards director i is a member

$$compliance_i = \frac{\sum_j X_{ij}\, cost_j}{\sum_j X_{ij}}$$

The cost of board j is the minimum number of steps that board j should make to comply with the rule. A step is a change in the gender of a director.

The number of male and female director on a board were calculated constructing first the bipartite adjacency matrix, whereby X is a matrix of size NxM, where N is the number of boards and M the number of directors. X_{ij} is 1 of director j belongs to board i, and 0 otherwise. The variable $gender$ is a vector where $gender_i = 1$ if director i is a male and 0 if it is a female. Using these definitions, we can count the number of female and male directors by the following:

$$males_i = \sum_j X_{ij}\, gender_j$$

$$females_i = \sum_j X_{ij} - males_i$$

2. Homophily

The most important sociological category for this analysis is homophily. We utilize the network concept of 'modularity' found in the community detection literature (e.g. Newman, 2001). We calculate the preference for homophily as the deviation from the expected gender proportions given the percentage of female and male directors. The second term in the expression below relies on the operator of the exclusive or (xor) to count the number of same gender ties proportions as a proportion of all ties. If the empirical value is equal to the empirical expected value, then there is no homophily and the value of the expression is 1. The quota will increase the homophily in the network (seen as a deviation from value of 1), as the preference for women increase due to compliance.

$$homophily_i = 1 - \frac{\sum_j (XX^t)_{ij} xor\left(gender_i, gender_j\right)}{\sum_j (XX^t)_{ij}} - Expected\ value$$

3. Popularity

Popularity of a director is measured simply by the log of the degree, i.e. the number of boards to which a director belongs.

$$popularity = \log \sum_j X_{ij}$$

4. Male and Female clustering coefficients

Since we are matching directors and boards, the graph is bi-partite and the usual cluster coefficients created for unipartite, or projections of the bipartite graph do not apply. We rely upon an idea in Robins and Alexander (2004) to calculate the appropriate bipartite cluster. For the projection, we can measure the degree of clustering as the proportion of closed triangles (times three) over the number of open triangles (that is, two stars whose focal node belongs to the triangle). Robins and Alexander propose the same idea for measuring closure for a bipartite network. Opsahl (2011) proposed a more direct way to calculate the bipartite clustering coefficient as the ratio of 4-paths and all possible closed 4 paths:

$$C *= \frac{closed\ 4\ paths}{possible\ 4\ paths}$$

The values for this clustering coefficient range from (0,1) and equal to 1 for a fully connected graph. The average value is found by counting the numerator and denominator over all cycles.[20] We calculate the average clustering coefficients separately for male and female directors, since past studies reviewed above indicate that women often lack cohesive local groups.

5. Random value

The random utility functions include, by definition, a random component; a uniform distribution is used to generate a value bounded by (0,1).

In summary, the utility function consists of six components: compliance, homophily, popularity, male and female clustering coefficients, and a random value.

Genetic Algorithms and Fitness Function

We need to find reasonable weights to use in the utility functions of directors to determine if they will offer, or accept, the invitation to the board.[21] Whereas the above statistics can be calculated from the data, the weights are unknown and must be found. As

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[20] Opsahl's publication came to our attention after we had implemented the same algorithm.

[21] An earlier simulation applied to Belgian data compared rules (see Kogut, et. al., 2012); GA permits an easier way to compare quotas.

the US has not imposed quotas, we do not know how the quota itself influences the choice. However, we do have observations on the Norwegian experiment. We therefore first find the weights for Norway, and then use these for the US simulations.

The technical challenge is the following. Given two networks N_0 and N_1 that are consecutive in time, we must find the weights of the utility function such we can generate from N_0 a N_1 that best approximate the characteristics, or features, observed in the empirical network. For networks of even moderate size, the computational cost is too high to use a brute force or classical statistical approach and we use therefore genetic algorithms to find the weights that provide the best fit to the empirical data.

Genetic algorithms (GA) employ a search heuristic for difficult optimization problems by mimicking natural evolution in regard to four properties: inheritance, selection, mutation and crossover. An agent is characterized by a genetic string or chromosome, which in our case are the weights to the utility function. In general, a genetic algorithm begins by generating a random population of individuals, or agents. The best fitted individuals are selected to give offspring to the next generation. GA uses two operators to generate new solutions from existing ones: *crossover* and *mutation*. The crossover operator is the most important operator. In crossover, two chromosomes, called *parents*, are combined to form new chromosomes, called *offspring*. With some probability, a gene in a sequence is mutated. This evolutionary process is repeated for N generations and evaluated against the improvement in population fitness. Those sequences (or solutions) that fail to meet a fitness threshold are eliminated, whereas successful sequences are retained.

The fitness function is defined by a feature vector of a network consisting of 5 properties:

1. *homophily*: propensity of members of the same gender to be linked together

2. *gender ratio*: number of females/number of directors

3. *degree ratio*: number of links with a female on one end/number of links

4. *average degree*: average degree of the directors

5. *giant component*: the largest connected component of the bipartite graph

We would like to find weights to the utility function such that the generated network N_1 provides a satisfactory fit to the empirical values of these five properties. More precisely, the GA searches for the weights that minimize the Euclidean distance between the two vectors of the simulated network and the empirical network.

In summary, we follow a strategy of creating a population of 16 sequences and then applying the genetic algorithm described above over 50 generations. Each sequence is a vector consisting of the six weights used in the utility functions. Using the observed data (N_0), we grow the subsequent network by a simulation using a fixed vector of weights, and evaluate the fit to the observed network. Then applying the rules of crossover and a mutation rate of .2, we generate a new population of sequences. We do this for the 50 generations, thereby improving the fitness of the weights. We repeat this analysis again for the next pair of years, and find again the best weights. We chose to average the best fitted weights across the years; these are the weights that are then applied to the simulations using the US data for 2007.

The Optimal Weights

Table 3 provides the results of finding the weights by applying a genetic algorithm to the Norwegian data as explained earlier. The first row gives the arguments to the utility function, indicating for example that a director cares about homophily, or increasing his or her popularity, or choosing directors that belong to its neighborhood cluster. The final argument allows for random error. These weights are then used in the US simulation, to which we now turn.

7. Evaluating Quotas for the United States

Since we cannot observe the weights to the arguments in the utility functions, we estimate first the weights on the Norwegian data. Because Norway imposed quotas, the empirical observations on the evolution of the Norwegian board networks overtime provide a useful empirical grounding to set the weights to the utility function. Then using these weights, we simulate a matching market whereby directors join boards, using the US 2007 board-director network and requiring that the boards comply in five years.

The US Simulation Results

The simulation results are graphed for the key network statistics we discuss above in figures 3a to 3f. A consistent pattern across the panels is the implied criticality of the quota between 10% and 20%. Raising the quota to 20% crosses a threshold for most the statistics, indicated by the divergence in the evolution of the key statistics.

Figures 3a to 3f summarize the key results from the simulations using the US data. The first panel shows the degree of compliance to the quota. The lower compliance for the 40% quota is not surprising, as the lower quotas are almost met by the initial empirical boards. It is useful to consider what a quota achieves. The percentage of female directors for the US in our data is just around 8.4. Though close to 10%, requiring a quota in this case still has meaning, since some boards have a high percentage and some boards have zero female directors, as we showed earlier. In this case, the primary outcome is to require that all boards comply with the rule. Of course, there is a so-called ‘integer’ problem with a quota, especially for small boards. For example, a board of 7 will be required to have 1.4 female directors if the quota is 20%. The Norwegian law therefore stipulated precisely when compliance should be rounded up and down depending on board size. A board can be in compliance even if it does not strictly mean the percentage requirement. For many boards, the compliance was not met due to these integer constraints and also the costliness to fully comply.

The second panel given in figure 3b shows the results for homophily. Homophily is measured by the gender modularity of the graph. A number greater than 0 means that the directors are grouped together by gender and there is positive segregation, while negative numbers mean that they are mixed together. The higher the quota we impose, the lower the segregation will be. Our imposed 'rewiring' of the graph without the quota leads to increased segregation, which indicates the baseline tendency of male boards to invite males, and more female boards to invite females. This is a nice baseline for the simulations.

As explained above, an important nodal property of a graph is the degree. Figure 3c takes the ratio between women and men directors; the degree falls in the case of no quota, indicating that female directors tendency to be invited onto multiple boards falls in the absence of a quota. The 20% quota indicates a slight improvement and this trend is, not unsurprisingly, increases in the quota. Since we estimated the parameters to the utility function using the Norwegian data which had no entry or exit of directors by our construction, we also did not account for turnover in the US case; directors can become isolates –waiting to be rewired, and isolates can be recruited. This construction simplifies the simulation. Obviously, for high quotas, the female degree will increase. However, this increase is governed by the estimated parameters to several rules (such as popularity or clustering), and it will not necessarily lead to highly central women. As important, our main concern is to check for criticality as quotas increase, which will not be linear in degree.

The central result concerns then the comparison of the effects of different quotas on achieving structural equality in gender. We proposed that betweenness centrality is the appropriate measure, for it indicates the strength of brokerage played by a director. It is the absence of this brokerage that past research has found to explain the reliance of female managers to rely upon male colleagues and to achieve sufficient power to self-organize. Figure 3d indicates that betweeness centrality increases strongly for women relative to men for quotas 20% or higher. For the US case, a quota of 20% (half the mandated 40% imposed) in Norway achieves substantial structural equality.

Centrality applies to graphs that are connected, and it is useful therefore to look at the connectivity of the global networks and the female subgraph. Figure 3e shows the evolution of the giant component for each of the quotas. Overall, the graphs' connectivity of the overall network increases, since disconnected and new directors join the giant component through the rewiring; this effect is strongest for no and low quotas. However, for a quota of 40%, the imposed replacement of male by female directors leads to more rapid dissolution of connectivity and the size of the giant component falls.

Figure 3f illustrates this dynamic more clearly, whereby high quotas lead to more connected female subgraph (the network once male directors are removed). In comparison to the overall graph, high quotas increase the female giant quota. It would be surprising if the doubling and trebling of female directors should not have this effect. What is striking is that a 20% quota still more than doubles the connectivity of the female graph.

Criticality

The results of the simulation point to a critical threshold for a quota between 10% and 20%. To see if the network properties persist, we removed the quotas after 5 rounds (which were equivalent to 5 years). The drawback to very low quotas is the low connectivity among female directors. It is striking that for quotas 20% and higher, women retain structural equality in centrality. In comparison, in the absence of a quota, the position of women worsen; and for a quota of 10%, there is little change in the structural equality of women. This suggests that the quotas improve dynamically the position of women by increasing their clustering and connectivity. Without this improvement, the position of women decline given the weights estimated from the Norwegian data.

This sub-criticality relies entirely on a *structural* analysis. The pressure to increase female representation on boards is influenced, as many studies show, also by changing attitudes towards women leaders. In the analytical framework of our model, this tendency could be could be captured somewhat clumsily by increasing the weights and thus the utility to choosing high degree and clustered female directors; attitudes, captured by weights, rather than quotas drive social change. There is then an implication often noted: broader changes in attitudes towards female top managers and directors are required to achieve the conditions of sustained equality in the absence of quotas. The process of social and cultural change may be slower than anticipated unless coupled with political power through the creation of strong brokerage roles for women top managers by the imposition of mandated quotas.[22]

Conclusions

The path-breaking study by Kanter argued that patterns of sex segregation in

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[22] It is sobering that the study by Beaman et al., 2009, on political quotas for women in Indian villages found that attitudes toward the capabilities of women leaders improved, but the bias for male leaders did not substantially weaken. Somewhat related, Dahl et al. (2011) found companies where the CEO has a daughter provides higher wages to employees, but more so to males.

managerial ranks are unlikely to change dramatically through endogenous social processes -there was insufficient critical mass of women in the top ranks of corporations. In the absence of such processes, system-level phenomena require system-level intervention: the exogenous imposition of quotas to attain a relative number of women sufficient to tip a system to equality.

The simulation results for the US reached a fundamental conclusion. Assuming a social process that relies on selection among neighbors, relatively small changes in the numbers of exogenously mandated women can be effective in creating the conditions for increasing structural equality. A nice way to understand the underlying mechanics in a random setting is to take a gambler's perspective on the odds ratio. At 15% women to men directors, the odds ratio of a women director is a bit less than 7:1. At 20%, the probability is 5:1, and at 25%, it is 4:1. In other words, marginal changes of 5% in the female-to-male ratio result in decreasing marginal changes in the odds ratios (from a marginal change of 2 to a change of 1 in the example). Thus, the largest contribution to altering the structural properties of boards is gained when there are low percentages of women board members. Low numerical quotas are sufficient, by our simulations for the United States case, to generate a network of highly central and influential women directors.

However, in the absence of a social network or in very low female connectivity, low quotas will not be effective. Norway is a case in which the director network was in general fairly anomic. The process by which female directors were added favored slightly relying upon incumbent female directors, thus raising at first their centrality ratios. Given these social rules and low critical mass, a high quota was most likely required to improve both the numerical and structural equality of female directors.

In the case of US and similar countries whose director networks evidence modest to high connectivity, a numerical target of 20% appears to permit a fundamental structural change that permits the potential for a creation of a more egalitarian society at the top of the corporation. This result depends upon the decision of many firms to move to and beyond this level -that is, upon the creation of a critical mass of well connected female directors. At a modest numerical target, it is possible that normative pressures will be effective in the United States, without the need to stipulate external (and illegal) quotas. However, as discussed above, quotas without these normative changes are unlikely to lead to self-organizing tendencies in the absence of greater connectivity.

Our analysis cautions public policy that places a higher weight on achieving costly numerical parity than on structural equality. *De jure* quotas are illegal in the US and *de facto* quotas encounter a backlash (Sturm, 2006). In practice, the consideration of diversity as a factor in selection, be it students or directors, implies an implicit notion of minimal representation, which defines implicitly a quota. The purpose of a quota, we have suggested, is not to impose the consequences, but to impose the opportunity for social justice.

The analysis of this paper suggests then two observations. The first is that policy strategy recommendations can be informed by simulating possible worlds, guiding the choice regarding the range of the policy interventions (i.e. the percentage quota) and also the robustness of the choice to changes in parameters. The second is that for complex social networks, there is strong caution to extrapolate from one context to another. In the case of director quotas, the success or failure of policies in one country may often be poor guides to the desired intervention in another country. Initial conditions and the topological properties of the network (e.g the average degree) vary among countries, or policy settings, requiring attention to the specificities of each context.

All the more reason that simulations that bootstrap from empirical data can play a useful role in informing the choice and design of policies to achieve possible worlds. If firms are to influence public policies through analytical arguments, simulations, and more narrowly agent-based modeling, are useful in informing the debate on important issues, such as a central concern in corporate governance, namely the diversity among directors. In the current context of the explosion of data on the relationships among people in social networks, or the positioning of firms in cognitive and symbolic graphs, the attention to how small differences may lead to large changes renders possible worlds more accessible than suggested by linear comparisons. In the context of improving structural equality at the top of the corporation for women and minorities, the implication is that a small quota can achieve large structural consequences and such a policy is deserving of consideration.

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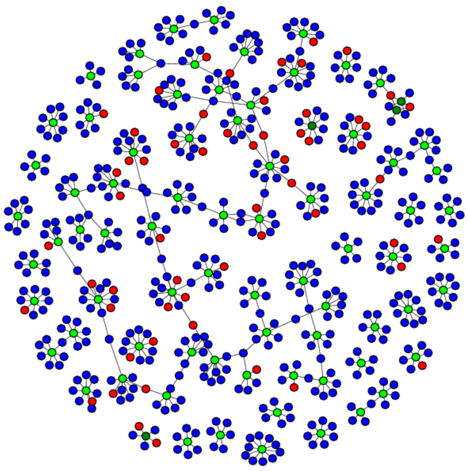
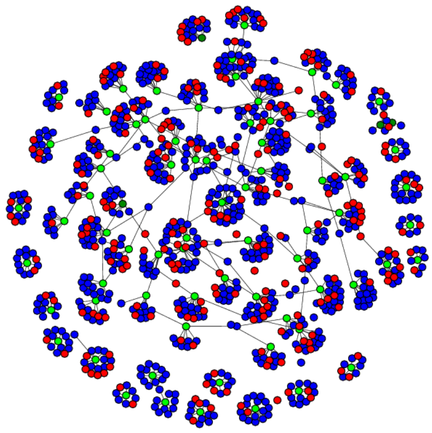
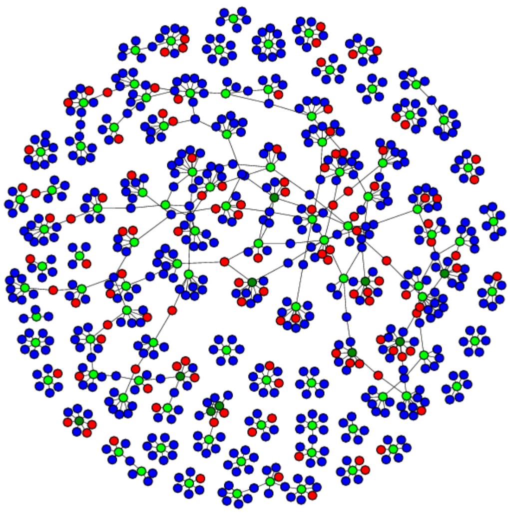
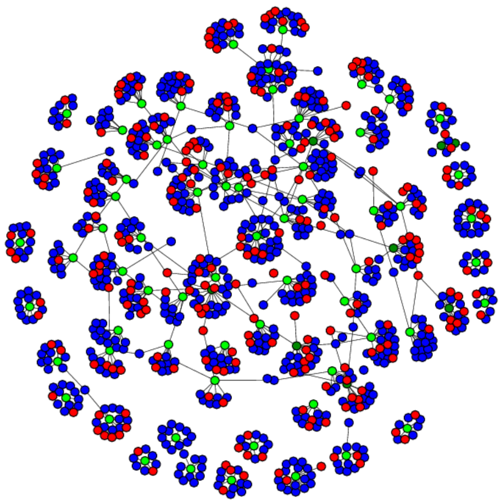
Figure 1

Evolution of the Norwegian Bipartite Director/Board Network for Select Years

Unfiltered	Filtered
a. 2003 	b. 2003 
c. 2005 	d. 2005 
e. 2007	f. 2007

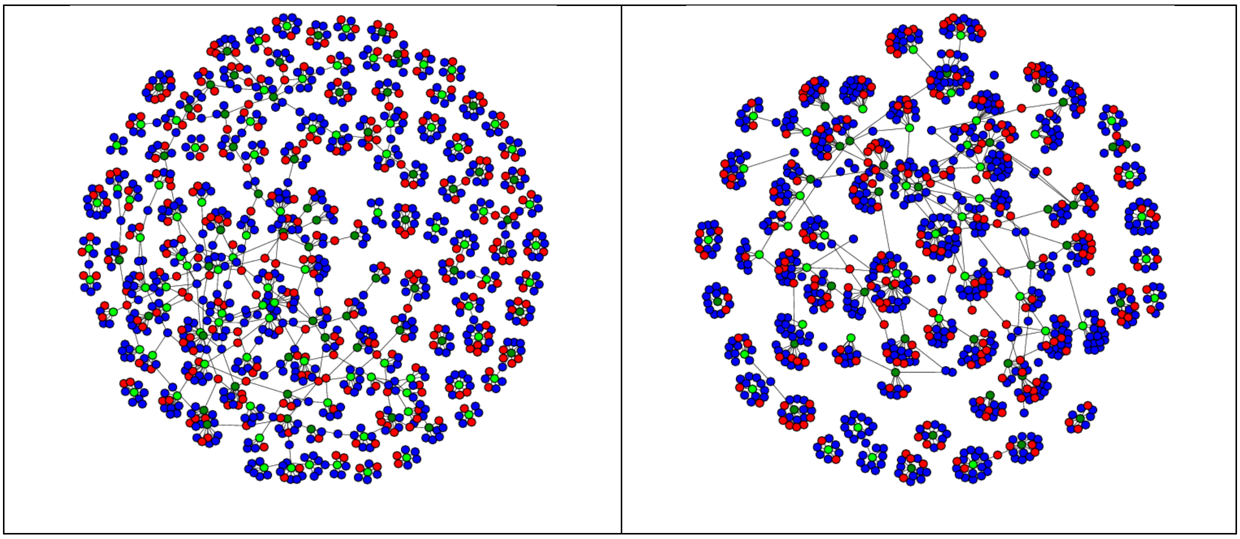


Figure 2

Histograms: U.S. Board Size and by Female Directors per Board

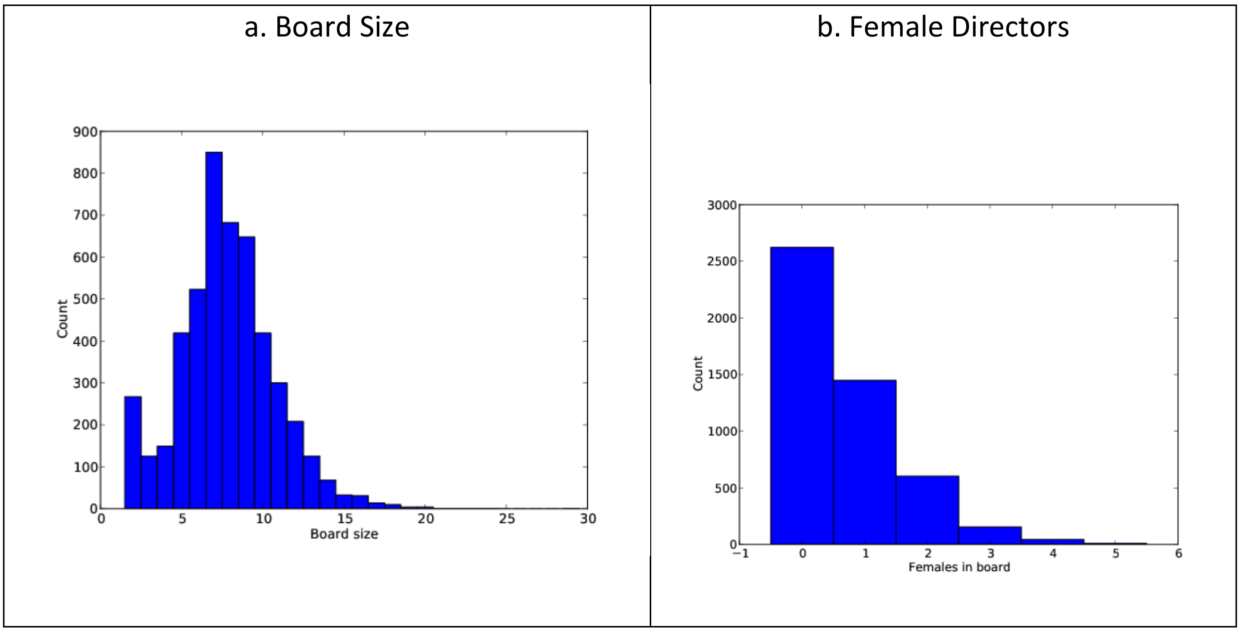


Figure 3

Simulation Results for the United States Board-Director Network and its Properties

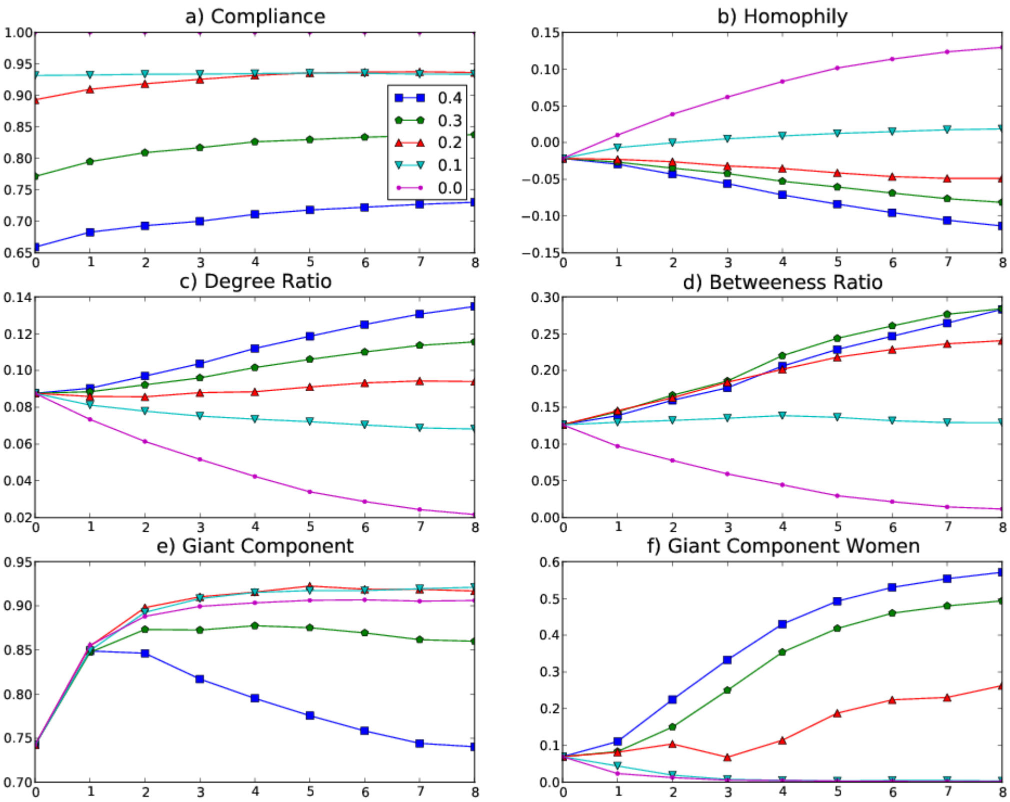


Table 2. Empirical Data for the Board-Director Networks

a. Norwegian Network Properties

year	firms	males	females	firms k	males k	females k	edges	density
2003	79	464	58	7.2	1.07	1.17	569	0.0138
2004	89	501	77	7.19	1.09	1.16	640	0.0124
2005	104	539	121	7.14	1.11	1.18	743	0.0108
2006	244	726	138	3.95	1.1	1.18	965	0.0046
2007	286	876	239	4.18	1.07	1.06	1196	0.0038
2008	248	620	236	3.82	1.09	1.13	949	0.0045

b. Network Properties for US, Sweden, Belgium, and India

country	year	firms	males	females	firms k	males k
Sweden	2001	191	1098	83	7.2723	1.1803
Sweden	2002	209	1150	90	7.1962	1.2148
Sweden	2003	210	1082	131	6.9714	1.2116
Sweden	2004	217	1109	189	7.3825	1.2227
Sweden	2005	219	1062	200	7.0457	1.2241
Sweden	2006	367	1740	316	6.8638	1.2253
Sweden	2007	276	1407	291	7.6812	1.2466
USA	2007	4884	27183	2487	7.8264	1.283
India	2007	23242	11138	584	2.2983	4.5825

Table 3. Fitted Weights to the Utility Function using the Norwegian Filtered Network

Compliance	Homophily	Popularity	CCM	CCF	Random
0.1802	0.5348	0.5575	0.3178	0.429	0.2326

CCM: clustering coefficient for male directors; CCF: clustering coefficient for female directors; random is the probability of a mutation.